# Comment on "Acoustics of tachyon Fermi gas" [E. Trojan and G.V. Vlasov, ArXiv: 1103.2276]

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In contrast to Trojan and Vlasov [1], it is found that an ideal Fermi gas of tachyons has a subluminous velocity of sound for any particle density and, therefore, the causality condition for a tachyon gas holds always true. Also, an ideal Fermi gas of tachyons never possesses an exotic equation of state with the pressure exceeding the energy density.

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Recently, Trojan and Vlasov [1] have reported the acoustic properties of tachyon Fermi ideal gas at zero temperature. In this study they assert that the Fermi gas of free tachyons, i.e., particles with the energy spectrum

$$\varepsilon_k = \sqrt{k^2 - m^2}, \quad (k \geqslant m),$$

should be unstable if the particle density $n$ of a gas is below some critical value $n_T$ because its sound velocity $c_s$ exceeds the speed of light, and thus the causality principle cannot be satisfied. In addition, authors [1] have also arrived at the next assertion that the gas of tachyons may represent an example of the so-called stiff matter when the pressure $P$ exceeds the energy density $E$, i.e., $P > E$.

Both the statements are erroneous. The false conclusions result from using incorrect equation (4) from [1]

$$P = \frac{\gamma}{6\pi^2}\int \frac{\partial \varepsilon_k}{\partial k} f_k k^3\, dk,$$

resulting from inaccurate integration by parts of the thermodynamical potential $\Omega$. Instead the authors [1] must use the starting relation which comes from the definition [2] of thermodynamical potential $\Omega = -PV$ leading to

$$P = \gamma T \int \frac{d^3k}{(2\pi)^3}\, \ln\bigl(1 + e^{(\mu - \varepsilon_p)/T}\bigr),$$

where $\mu$ is the chemical potential and $\gamma$ is the degeneracy factor. For $T = 0$, the case of interest in [1], one has

$$P = \gamma \int \frac{d^3k}{(2\pi)^3}\,(\varepsilon_F - \varepsilon_k) = n\varepsilon_F - E,$$

where $\varepsilon_F = \mu(T=0) = (k_F^2 - m^2)^{1/2}$ is the Fermi energy and $E$ is the energy density of a gas. Using $E$ from [1],

$$E = \frac{\gamma m^4}{16\pi^2}\bigl[\bigl(2\beta^3 - \beta\bigr)\sqrt{\beta^2-1} - \ln\bigl(\beta + \sqrt{\beta^2-1}\bigr)\bigr],$$

gives another result for the pressure of a tachyon Fermi ideal gas at $T = 0$

$$P = \frac{\gamma m^4}{16\pi^2}\bigl[\bigl(\frac{2}{3}\beta^3 + \beta - \frac{8}{3}\bigr)\sqrt{\beta^2-1} + \ln\bigl(\beta + \sqrt{\beta^2-1}\bigr)\bigr].$$

In addition, the above expression agrees completely with the determination of the pressure as a derivative of the total energy $EV$ over gas volume $V$ with the opposite sign

$$P = -\partial(EV)/\partial V = n\partial E/\partial n - E.$$

The square of the sound velocity $c_s$ is then given by

$$c_s^2 = \partial E/\partial P = \frac{1}{3}\frac{\beta^2 + \beta + 1}{\beta^2 + \beta},$$

where $\beta = k_F/m \geqslant 1$ is a ratio of the Fermi momentum to the tachyon mass. Thus, unlike [1], the sound velocity, changing between $1/\sqrt{2}$ and $1/\sqrt{3}$, is always smaller than the speed of light and the causality principle $c_s < 1$ does not break down for any density of ideal tachyon gas.

As it concerns the ratio of pressure to energy density

$$\frac{P}{E} = \frac{1}{3}\frac{\bigl(2\beta^3 + 3\beta - 8\bigr)\sqrt{\beta^2-1} + 3\ln\bigl(\beta + \sqrt{\beta^2-1}\bigr)}{\bigl(2\beta^3 - \beta\bigr)\sqrt{\beta^2-1} - \ln\bigl(\beta + \sqrt{\beta^2-1}\bigr)},$$

it varies from 1/2 in the low density limit to 1/3 for the high density limit. As a result, the hypothesis [1] that the low density gas of free tachyon can represent an example of the so-called absolute stiff or hyperstiff matter with the equation of state $P \geqslant E$ proves to be invalid as well.

In [3] an attempt is made to resolve a confusion of Ref. [1] with the well-established thermodynamic relations by using the following energy spectrum of a tachyon

$$\varepsilon_k = \begin{cases} 0, & k < m \\ \sqrt{k^2 - m^2}, & k \geqslant m. \end{cases}$$

However, such energy spectrum does not satisfy the relativity principle that the square of four-vector $(\varepsilon, \boldsymbol{k})$ must be an identical scalar within the whole region of permissible $\boldsymbol{k}$, i.e.,

$$\varepsilon_{\boldsymbol{k}}^2 - \boldsymbol{k}^2 = \text{const}.$$

Obviously, this condition breaks down for the spectrum proposed.

In conclusion, the arguments in [1, 3] to consider a tachyon ideal gas at sufficiently low density either as an unstable $c_s \geqslant 1$ matter or as an exotic $P \geqslant E$ matter have no theoretical grounds.

[1] E. Trojan and G.V. Vlasov, ArXiv: 1103.2276; Phys. Rev. D **83**, 124013 (2011).
[2] L.D. Landau and E.M. Lifshitz, *Statistical Physics*, Part I, Third edition (Pergamon, Oxford, 1980), § 53.
[3] E. Trojan, *Careful calculation of thermodynamical functions of tachyon gas*, ArXiv: 1109.1026.